On the Relationship between Interest Rates and Volatility Regimes in Daily Stock Returns

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1. Introduction

The relationship between stock market volatility and the economy is a recurring theme in empirical finance research. Understanding the nature of this relationship is important to efforts in developing richer classes of asset pricing models. These models may also provide more accurate volatility forecasts than pure time series models for use in option pricing, hedging, and the timing of informed trades (see e.g., Kyle 1985, and Admati and Pfleiderer 1988). Despite extensive research on this subject, the quest for economic variables that can explain most of the variation in aggregate stock returns remains an illusive one. After analysing a century of monthly data, Schwert (1989) expresses “surprise that stock volatility is not more closely related to economic activity” (Schwert 1989, p. 1146). Schwert’s basic conclusion is echoed by many other studies that attempt to relate variations in stock returns to economic fundamentals. These empirical research typically find that variables that commonly proxy for economic fundamentals such as industrial output, GNP, dividend yields, short term interest rates, term spread and default spread explain a small fraction of the variation in stock returns. Moreover, explanatory power decreases as the return horizon shrinks. For example, using a subset of these proxies as explanatory variables for the CRSP value-weighted index, Whitelaw (1994) find $R^2$ of 41.2% for annual returns, 17.6% for quarterly returns and only 7.4% for monthly returns. Cheung, He and Ng (1997) find that only 8% of the variation in monthly real returns to the U.S. stock market can be explain by global economic variables that proxy for changes in discount rates and expected cash flows.
In this study, we take a different approach to investigating the information content of fundamentals in explaining stock volatility. First, we model the relationship between fundamentals and daily stock returns volatility. Previous research on the link between stock returns and the economy focused on monthly or lower frequency data primarily because of economic data constraints. In particular, data on industrial output and gross national product are only available on a monthly or quarterly basis. However, from a practical viewpoint, daily returns are of greater interest to investors wishing to take bets on or hedge against large moves in stock prices. Moreover, informed traders would like to camouflage their transactions by trading when trading volume is high. Given the contemporaneous correlation between trading volume and volatility (see Karpoff 1987), high-volume days usually occur in periods when volatility is high.

Second, recognizing that daily returns can be highly noisy, we increase the power of our tests by focusing on volatility regime shifts i.e. large and persistent jumps in volatility\(^1\). We model regime shifts using the Markov switching framework pioneered by Hamilton (1989). In our model, proxies for fundamentals drive the transition probabilities between two states: a high volatility state and a low volatility state. Thus, our model is a two-state Markov switching model with time-varying transition probabilities. We use proxies the following variables which are observable at the daily interval: 3-month Treasury bill rate, default spread (yield spread between Baa-rated and Aaa-rated bonds) and term spread (yield spread between 30-year Treasury and 3-month Treasury bill). These variables are chosen because they have theoretical

\(^{1}\) Daily returns are noisier than longer horizon returns in the following sense. First, a large fraction of daily trading volume and volatility seems to be noise- and liquidity motivated rather than news-driven (see Andersen 1996). This fraction is even higher for portfolios than individual stocks because in portfolio, the effects of firm-specific news are diversified away. Second, the time series of some fundamental variables
motivations, are observable at the daily interval and have demonstrated predictive power for monthly expected returns or volatility (see e.g., Keim and Stambaugh (1986), Fama and French (1989), Campbell (1987) and Whitelaw (1994). That these variables have been analysed extensively in previous studies does raise concerns about data snooping. To mitigate this concern, we test the stability of the model over different sub-periods and also its out-of-sample forecasting ability over different holdout periods. Our results show that the model is robust across different sub-periods and consistently delivers superior volatility forecasts compared to a single regime model and a Markov switching model with fixed transition probabilities.

This rest of this paper is organized as follows. Section 2 briefly describes the Markov switching approach in modelling regime shifts, the model specification used in this paper and the definitions of the financial variables. Section 3 describes the data and discusses estimation issues. Section 4 present estimation results for the full sample period and results of the forecasting experiment. Sub-period estimation results are reported in Section 5. Section 6 concludes the paper.

2. Markov Switching Specification

Markov switching models have been used to model nonlinearities in asset returns where the nonlinearities arise from discrete jumps in the conditional distribution between a finite number of regimes. The law of motion governing the switch between regimes is usually a first-order Markov process with constant transition probabilities. An appealing feature of Markov switching models is that the regimes need not be observable by the

such as dividend yields, industrial production and GNP have slow-varying components. Changes in these variables are reflected more fully in longer horizon returns than in daily returns.
a product of the estimation process. Hamilton (1989) has derived a nonlinear recursive filter that can be used to construct the log-likelihood function for maximum likelihood estimation.

In this paper, we estimate a Markov switching model with time varying transition probabilities that are driven by proxies for economic fundamentals. To keep the model tractable, we restrict the number of regimes to two. The basic structure of the model is as follows. Let \( r_t \) be the rate of return for period \( t \). We characterize the conditional mean return as a simple \( J \)-order autoregressive process:

\[
    r_{it} = \alpha_i + \sum_{j=1}^{J} \beta_{ji} r_{t-j} + \varepsilon_{it}
\]

where \( j = 1, 2 \) denote the two states \( (S_j) \), and \( \varepsilon_t \) is distributed with mean zero and conditional variance \( h_{it} \). Let \( f_{it} \) denote the distribution of returns, conditional on the \( i \)th regime and available information at \( t-1 \). Assuming conditional normality for each regime, we can write the conditional distribution for the first regime, \( f_{1t} \), as follows:

\[
    f \left( r_t | S_t = 1, \Omega_{t-1} \right) = \frac{1}{\sqrt{2\pi h_t}} \exp \left\{ -\frac{1}{2} \frac{(r_t - \mu_{1t})^2}{h_{1t}} \right\}
\]

where

- \( \mu_{1t} \) = conditional mean return at time \( t \)
- \( h_{1t} \) = conditional variance of returns at time \( t \)
- \( \Omega_{t-1} \) = information set at time \( t-1 \)

For simplicity, no dynamics are incorporated in the conditional volatility. This may appear to be restrictive for daily returns given the evidence on time varying e.g. GARCH-type volatility. However, Lamoureux and Lastrapes (1990) and Hamilton and Susmel (1994) note that the degree of GARCH volatility persistence often drops considerably
after accounting for regime shifts in the volatility process, indicating that the higher degree of volatility persistence implied by GARCH models are a statistical artefact induced by structural breaks in the data\(^2\).

The transition between regimes is assumed to be directed by a first-order Markov process with the following transition probability matrix:

\[
P_t = \Pr(S_t = 1|S_{t-1} = 1)
\]

\[
1-P_t = \Pr(S_t = 2|S_{t-1} = 1)
\]

\[
Q_t = \Pr(S_t = 2|S_{t-1} = 2)
\]

\[
1-Q_t = \Pr(S_t = 1|S_{t-1} = 2)
\]

Let \(\Pr(S_t = i | \Omega_{t-1})\) denote the regime probability that regime \(i\) is in operation at time \(t\), conditional on the available information set. From Gray (1996), it can be shown that the regime probability \(p_{1t}\) can be written as a non-linear recursive function of the transition probabilities and the conditional distribution:

\[
p_{1t} = P_t \left[ \frac{f_{1t-1} p_{1t-1}}{f_{1t-1} p_{1t-1} + f_{2t-1} (1-p_{1t-1})} \right] + (1-Q_t) \left[ \frac{f_{2t-1} (1-p_{1t-1})}{f_{1t-1} p_{1t-1} + f_{2t-1} (1-p_{1t-1})} \right]
\]

where

\[
p_{1t} = \Pr(S_t = 1|\Omega_{t-1})
\]

\[
f_{1t} = f(r_t|S_t = 1)
\]

\[
f_{2t} = f(r_t|S_t = 2)
\]

\(^2\) Timmerman (2000) shows that regime-switching models can generate a rich array of data characteristics, including serial correlation in levels, volatility clustering, skewness and kurtosis. He finds that even simple low-order regime-switching models represent a very flexible tool for modeling the complex dynamics of financial time series.
Given the above set-up, the model can also be interpreted as a mixture-of-normals model with the regime probability as the mixing variable. To investigate whether financial variables explain regime shifts, we allow the transition probabilities to depend on the selected economic variables as follows:

$$P_t = \phi \left( c_1 + d_{A,1} DS_{t-1} + d_{B,1} TS_{t-1} + d_{C,1} TB_{t-1} \right)$$

$$Q_t = \phi \left( c_2 + d_{A,2} DS_{t-1} + d_{B,2} TS_{t-1} + d_{C,2} TB_{t-1} \right)$$

$$p_{1,t} = (1 - Q_t)\Pr(S_{t-1} = 1|\theta_{t-1}) + (P_t)\Pr(S_{t-1} = 1|\theta_{t-1})$$

where \(\phi(.)\) is the cumulative normal distribution function to ensure that \(Q_t\) and \(P_t\) lies inside the unit interval, \(DS_t\) is default spread, \(TS_t\) is term spread and \(TB_t\) is Treasury bill rate. These financial variables were chosen because they have been used extensively in studies of equity risk premia and volatility e.g. Campbell (1987), Keim and Stambaugh (1986), Fama and French (1989), Schwert (1989), Glosten, Jagannathan and Runkle (1993), Whitelaw (1994) and Pesaran and Timmerman (1995). There is a valid concern that our choice of financial variables may lead to data snooping biases. However, previous studies used mostly monthly data, where the universe of possible explanatory variables is much larger than is the case of daily data. In other words, the scope for data snooping is arguably smaller in our study than in studies based on monthly data. Nevertheless, to allay concerns over data snooping biases, we estimate our model over different sub-periods and test its forecasting performance using different out-of-sample periods.

Default spread is measured as the yield spread between Baa-rated and Ass-rated long term corporate bonds. The default spread is a counter cyclical indicator of business conditions, rising when business conditions are weak and narrowing when business...
conditions are strong. This is not surprising since bond investors would demand a higher risk premium in periods when default risk is high. Schwert (1989) shows that even in the presence of other variables, the default spread is positively related to future stock market volatility. We expect the probability of a switch to the high volatility state to be positively related to the default spread.

Term spread is measured as the yield difference between 30-year Treasury bonds and 3-month Treasury bills. The term spread is a commonly used predictor of economic conditions. A larger term spread signals more robust economic growth while a smaller term spread signals slower growth. Consistent with this hypothesis, empirical evidence by Roma and Torous (1997) show that the U.S. term structure is steeper at business cycle troughs and flatter at business cycle peaks while Andreou, Osborn and Sensier (2000) provide similar evidence for the U.S., Germany and the U.K. Using monthly data, Keim and Stambaugh (1986) and Fama and French (1989) examine the linkage between term spread and subsequent month stock market returns. They find that a higher term spread generally leads to higher stock returns. No study has examined whether a similar linkage exists for daily returns.

Our third economic variable is short term interest rates as proxied by the yield on 3-month Treasury bills. Empirical evidence on the effect of short term interest rate on monthly stock volatility is mixed. Campbell (1987) shows that short-term interest rates are positively related to subsequent month stock volatility. More recent evidence allowing for nonlinearities in the relationship between conditional moments e.g. Whitelaw (1994) shows that short term interest rates have no significant effect on monthly stock volatility after controlling for default spread. No study has examined the
impact of changes in short-term interest rates on daily stock volatility. We conjecture a negative relationship between short-term interest rates and daily volatility. At the macro level, lower short-term rates may reflect an easier monetary policy that may induce expectations of a more robust economy. At the micro level, lower short-term interest rates may stimulate more liquidity-motivated and margin-sensitive “noise” trades (see Chordia, Roll and Subrahmanyam 2001). Andersen (1996) shows that over half of the daily volume for some large capitalization stocks is related to liquidity and uninformed trading, with the balance due to news arrivals.

3. Estimation Issues and Data

3.1 Maximum Likelihood Estimation

The log-likelihood function \( L \) for the Markov switching model is:

\[
L = \sum_{t=1}^{T} \log \left[ p_{1t} f_{1t} + (1 - p_{1t}) f_{2t} \right]
\]  

The log-likelihood function is constructed using the above recursion for the regime probabilities together with following expressions for \( h_t \) and \( \varepsilon_t \):

\[
h_t = E[r_t|\Omega_{t-1}]^2 - [E[r_t|\Omega_{t-1}]]^2
\]

\[
= p_{1t} (\mu_1^2_{1t} + h_{1t}) + (1-p_{1t}) (\mu_2^2_{1t} + h_{2t}) - [p_{1t}\mu_{1t} + (1-p_{1t})\mu_{2t}]^2
\]  

\[
\varepsilon_t = r_t - E[r_t|\Omega_{t-1}] = r_t - [p_{1t}\mu_{1t} + (1-p_{1t})\mu_{2t}]
\]  

The model is estimated using maximum likelihood based on the Broyden, Fletcher, Goldfarb and Shanno numerical algorithm as implemented in the GAUSS programming language (see Gill, Murray and Wright 1981). We start the optimisation process with different arbitrary initial values to ensure that the final estimation results are robust.
Standard errors for parameters are computed using White's (1980) heteroskedastic-consistent covariance estimator.

3.2 Data

The data for this study consists of daily returns of the Standards and Poor 500 stock index. The sample period is from January 3, 1986 to December 18, 2001 for a total of 3802 observations. Daily returns are computed as the natural log of daily price relatives. The raw price series is collected from Datastream and is adjusted for dividends and capitalization issues. We also collected data for the three interest rate instruments: 3-month Treasury bill rate, yield spread between Baa-rated and Aaa-rated long term corporate bonds (the default spread) and yield spread between 30-year Treasury bonds and 3-month Treasury bills (term spread). Data for the interest rate instruments were obtained from the FRED database located at the website of the Federal Reserve Bank of St. Louis.

4. Estimation Results for Full Sample

Table 2 presents the parameter estimates using data for the full sample period. The conditional variance appears to separate into two regimes. In particular, regime 1 is a high-variance regime and is associated mainly with positive average returns. In contrast, regime 2 is characterized low-variance, negative mean regime. The unconditional variance in the high volatility regime is 3.2 times the variance in the low volatility regime. The unconditional mean for the high volatility (low volatility) regime is 10.7% per annum (−2.57% per annum) respectively. These results are broadly consistent with

The model yields a maximized log-likelihood value of –1,001. For comparison, we also estimated a similar Markov switching model but constraining the transition probabilities to be constant. This restricted model yields a maximized log-likelihood values of –1,209. The likelihood ratio test easily rejects the restricted model in favour of the model with time-varying transition probabilities.

We also estimated a single regime model to evaluate whether allowing for two regimes describes the data better. The maximized log-likelihood value for the single regime model is –1,415. However, standard likelihood ratio tests are invalid for this test because under the null hypothesis of a single regime, parameters for the second regime cannot be identified which invalidates a key assumption necessary to justify the use of the likelihood ratio test. To adjust for this nuisance parameter problem, we adjust the p-values of the standard likelihood ratio test upwards using the method due to Davies (1987). Assuming that the likelihood function exhibits a single peak when viewed as a function of the $n$ nuisance parameters that are unidentified under the null, an upper bound on the significance level of the likelihood ratio test statistic is given by $P(\chi_T > 2L) + 2L^{T/2} [\exp^L \Gamma(T/2)]^{-1}$ where $\Gamma(.)$ denotes the gamma function, $L$ is the difference between the maximised log-likelihood under the alternative and null hypotheses and $T$ is sample size. Based on Davies test, we find that the adjusted p-value for the likelihood ratio test statistic for the regime-switching model is still well below 1%. This indicates that the regimes identified by the Markov switching model are not statistically spurious.
To assess the predictive power of the Markov switching model, we perform out-of-sample forecasting tests. Two benchmark models were used as comparisons. The first benchmark is a single regime model and the second benchmark is a two-state Markov switching model with fixed transition probabilities. The conditional mean for all models is an AR(1) process. The test proceeds as follows. First, we estimate the parameters of a particular model based on an in-sample period. Next, we use the estimated parameters to compute one-day-ahead volatility forecasts over a subsequent holdout period. Third, forecast errors are computed for each model as the difference between actual squared innovations and the volatility forecast. Finally, summary statistics to evaluate forecast accuracy are computed using the time series of forecast errors. Four evaluation criteria were used in this paper: mean square error (MSE), mean absolute error (MAE), mean percentage error (MPE) and $R^2$. MAE uses a linear penalty for forecast errors whereas MSE and MPE use a quadratic penalty. Which metric is more suitable depends on the loss function of the user. Since errors arising from forecasting large jumps in volatility can be very costly, the MSE may be a more appropriate criterion.

We perform two sets of forecast experiments. In the first set, the models were estimated from the start of our sample to December 31, 1992. The corresponding hold-out period for this experiment runs from January 4, 1993 to December 31, 1995. In the second set, the models were estimated from the start of the sample to January 2, 1997. The corresponding hold-out period for this experiment is January 3, 1997 to the end of the sample (January 18, 2001). The first hold-out period is a relatively stable period whereas the second hold-out period, which includes the Asian financial crisis and the
Russian debt crisis, is more volatile. The standard deviation of daily S&P 500 returns for these two periods was 0.00595 and 0.01266 respectively.

Table 3 report the forecasting results. Our Markov switching model outperforms both benchmarks on all four criteria over the two hold-out periods. The extent of superior performance can be readily seen in the last column which reports the ratio of performance measures for our model to those of the benchmarks. The single regime model performs worst among the three models, which is not surprising since the model will over- (under-) estimate the true variance following a large decrease (increase) in the variance. More interestingly, our model also performs much better than the simple Markov switching model in both hold-out periods, demonstrating the practical value of using the interest rate variables to explain shifts in volatility regimes. Forecasts generated from our model explains between 47% to 61% more of the variation in actual volatility compared to the simple switching model. The MSE of our model is only 61.5% to 66.3% of the MSE for the simple switching model. Ratios based on the other two forecast error criteria give a broadly similar picture.

Because forecast errors from the two models are contemporaneously correlated, it is not meaningful to apply a standard F-test to these ratios to determine which model yields statistically superior performance. Diebold and Mariano (1995) discuss simple alternative tests which are robust to contemporaneous correlations of forecast errors. Following the approach of Diebold and Mariano and assuming a quadratic loss function, we test the following null hypothesis: $H_0: E(e_3^2) = E(e_2^2)$ where e denotes forecast error and the subscript “2” and “3” denote Model 2 (Markov switching model with fixed transition probabilities) and Model 3 (Markov switching model with time-varying transition
probabilities) respectively. Let \( r_t = e_{2t} - e_{3t} \) and \( s_t = e_{2t} + e_{3t} \). Then, \( H_0 \) is equivalent to \( \text{corr}(r_t, s_t) = 0 \). Based on standard correlations tests, we are able to easily reject this null at below 5% for both hold-out periods. This implies that the model which has lower mean square error (Model 3) delivers statistically superior forecasting performance compared to Model 2. All in all, the forecast results indicate that our model is not overparameterized and is of practical use in forecasting large volatility jumps.

We now discuss the precise relationship between the interest rate variables and volatility regimes. First, Table 3 shows that all three interest rate instruments in the transition probability are significant across the two regimes. The coefficient for default spread \( (d_A) \) is 0.069 in the high variance state and -0.356 in the low variance state. Thus, a rise in the default spread increases the probability that stock returns will switch to or remain in the high variance state. Of the three interest rate variables, the coefficient for default spread has the largest absolute value, indicating greater explanatory power in predicting volatility switches compared to term spread and the bill rate. This result is consistent with that of Schwert (1989) who finds that even in the presence of other variables, default spread is positively related to future stock market volatility. See also Chen (1991).

The coefficient for the term spread \( (d_B) \) is 0.022 in the high variance state and –0.0047 in the low variance state. Both coefficients are significant at 1%. Thus, an increase in the term spread increases the probability that stock returns will switch to the high volatility state or remain in that state. This is a new result. Previous studies using monthly data only provide indirect evidence of a linkage between term spread and stock volatility. For example, Keim and Stambaugh (1986) find that over the period 1928-78,
the monthly risk premium for large capitalization stocks are positively and significantly related to term spread. Fama and French (1989) find some evidence (at 10% significance level) of a positive relationship between term spread and subsequent month stock returns for the period 1927-87, and stronger evidence in a later sub-period (1941-87). Our results complement these studies by showing that a term premium still exists in more recent times, even at the level of daily returns.

The coefficient of the bill rate ($\alpha_C$) is $-0.0173$ in the high variance state and $0.0045$ in the low variance state, with both coefficients statistically significant. Thus, lower short-term interest rates increase the probability of a switch from the low volatility state to the high volatility state. This is consistent with the hypothesis that lower interest rates raises expectations of a more robust economy. There may also be market microstructure reasons at work. For example, Andersen (1996) find that a large proportion of daily stock trading volume and volatility is unrelated to information flows but appear to reflect liquidity and noise trading. In contrast to informed trading, liquidity and noise driven trades are more sensitive to transaction costs (Davis and Norman 1990). Lower interest rates, by reducing net transaction costs, may induce more of such trades and therefore higher volatility.

5. **Sub-period Results**

To test whether our results are robust to time period, we re-estimated the Markov switching model for three equal sub-periods of 1158 observations each. The sub-periods are:
• Sub-period 2: January 8, 1991 – January 11, 1996
• Sub-period 3: January 12, 1996 – January 18, 2001

Table 4 reports the sub-period results. Qualitatively, the results for all three sub-periods are similar to the full sample results. For example, regime 1 is consistently the high mean, high variance regime while the opposite holds for regime 2. The ratio of high to low conditional variance ranges from 2.7 in the first sub-period to 5.4 in the second sub-period. As in the full sample, all three interest rate variables in the transition probability equation are statistically significant and have the same signs as before. Thus, the estimation results are robust across these different time periods.

6. Conclusion
The volatility of stock returns often fluctuate between regimes of high and low variance. Using a two-state Markov switching model with time-varying transitions, we examine whether observable economic variables can explain such regime switches using daily data. Consistent with previous empirical studies, we find strong evidence of regime shifts, especially in the conditional volatility of daily returns. The mean return is positive in high volatility periods and negative in low volatility periods. The results are consistent with a positive relationship between expected returns and risk.

Regime switches were found to correlate with interest rate instruments observable at the daily interval. Both the default spread and term spread are positively related to the probability of switching to the high variance state. These results are robust across different sub-periods and are consistent with past studies based on monthly data. We
also find that a negative relationship between short-term interest rate and the probability of switching to the high variance state. This is consistent with investors’ expectations of a more robust economy following periods of low interest rates. It may also reflect the fact that a large proportion of daily trading volume and volatility are related to liquidity and noise driven transactions.

Overall, despite the high level of noise in daily stock returns, this study demonstrates that observable economic fundamentals can explain significant changes in daily stock volatility. Our results thus complement existing studies focusing on data at the monthly or quarterly interval. Our Markov switching model also delivers superior short-term volatility forecasts compared to simpler models that ignore regime shifts and economic variables. The model is therefore of practical appeal to active investors who either wish to take bets on or hedge against large market moves.
References


Davies, R. B. (1987), "Hypothesis testing when a nuisance parameter is present only under the alternative", *Biometrika* 74, 33-43.


Glosten, L. R., R. Jagannathan and D. Runkle (1993), Relationship between the expected value and the volatility of nominal excess returns on stocks, Journal of Finance 48, 1779-1802.


Table 1. Estimates of Regime-switching Model with Time-varying Transitional Probabilities

The model is
\[ r_{it} = \alpha_i + \beta_i r_{i,t-1} + \varepsilon_{it} \]

where \( r_{it} \) is continuously compounded daily returns on the S&P 500 index, \( i = 1 \) or \( 2 \) denotes the state and \( \varepsilon_t \) is distributed with mean zero and conditional (on regime) variance \( h_i \). The transition probabilities of the states is given by:

\[ P_t = \Pr(S_t = 1 | S_{t-1} = 1) \]
\[ (1 - P_t) = \Pr(S_t = 2 | S_{t-1} = 1) \]
\[ Q_t = \Pr(S_t = 2 | S_{t-1} = 2) \]
\[ (1 - Q_t) = \Pr(S_t = 1 | S_{t-1} = 2) \]

where \( P_t = \phi \left( c_1 + d_{A,1} DS_{t-1} + d_{B,1} TS_{t-1} + d_{C,1} TB_{t-1} \right) \) and
\[ Q_t = \phi \left( c_2 + d_{A,2} DS_{t-1} + d_{B,2} TS_{t-1} + d_{C,2} TB_{t-1} \right) \]

\( DS \) is default spread (Baa-rated minus Aaa-rated long term corporate bonds), \( TS \) is term spread (30-year bond yield minus 3-month Treasury bill yield) and \( TB \) is the 3-month Treasury bill yield. The data is from Datastream and the sample period is from January 3, 1986 to December 18, 2001 (3802 observations). The t statistics are computed based on White’s (1980) heteroscedasticity-consistent estimator for standard error. * denotes the estimate is significant at 1%.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Parameter Values</th>
<th>t-statistics</th>
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<tbody>
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<tr>
<td>( \alpha_2 )</td>
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<td>( d_{C,2} )</td>
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</table>
Table 2. Out-of-Sample Forecast Tests

This table reports the forecasting performance of three models for the volatility of daily returns on the S&P 500 index. MSE is mean square error, MAE is mean absolute error, MPE is mean percentage error. Forecast error is measured as the difference between forecast and actual squared innovations. Model 1 is a single regime model. Model 2 is a two-state regime switching (RS) model with fixed transitional probabilities and Model 3 is a two-state regime switching model with time-varying transitional probabilities (TVTP). The conditional mean for the three models is an AR(1) process. The specification for Model 3 is described in the notes to Table 1. Parameters are estimated over the in-sample period and held fixed over the out-of-sample or hold-out period. Panel A reports results for the first hold-out period from January 4, 1993 to December 5, 1995 (740 observations), using data from January 3, 1986 to December 31, 1992 as the in-sample period. Panel B reports results for the second hold-out period from January 3, 1997 to January 18, 2001 (1020 observations) using data from January 3, 1986 to January 2, 1997 as the in-sample period. The last two columns report the ratio of MSE, MAE, MPE and $R^2$ for Model 3 to those of Model 1 and Model 2.

### PANEL A

#### Hold-out period 1: January 4, 1993 to December 5, 1995

<table>
<thead>
<tr>
<th>Evaluation Criterion</th>
<th>Model 1 Single Regime</th>
<th>Model 2 RS</th>
<th>Model 3 RS (TVTP)</th>
<th>Forecast evaluation ratios relative to</th>
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</tr>
<tr>
<td>MAE</td>
<td>0.0926</td>
<td>0.0545</td>
<td>0.0381</td>
<td>0.411</td>
</tr>
<tr>
<td>MPE</td>
<td>-0.3915</td>
<td>-0.3251</td>
<td>-0.2406</td>
<td>0.615</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.0153</td>
<td>0.1478</td>
<td>0.2179</td>
<td>14.24</td>
</tr>
</tbody>
</table>

### PANEL B

#### Hold-out period 2: January 3, 1997 to January 18, 2001

<table>
<thead>
<tr>
<th>Evaluation Criterion</th>
<th>Model 1 Single Regime</th>
<th>Model 2 RS</th>
<th>Model 3 RS (TVTP)</th>
<th>Forecast evaluation ratios relative to</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSE</td>
<td>0.7170</td>
<td>0.5948</td>
<td>0.3943</td>
<td>0.550</td>
</tr>
<tr>
<td>MAE</td>
<td>0.1915</td>
<td>0.1326</td>
<td>0.0552</td>
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<td>MPE</td>
<td>0.6610</td>
<td>0.5896</td>
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<td>$R^2$</td>
<td>0.0110</td>
<td>0.1328</td>
<td>0.2141</td>
<td>19.46</td>
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</table>
Table 3. Subperiod Analysis of the Regime Switching Model with Time-varying Transitional Probabilities

This table reports estimates of the Markov switching model with time-varying transition probabilities for three equal sub-periods as indicated below. The number of observations for the sub-periods are 1267, 1268 and 1267 respectively. The model specification is described in the notes to Table 1. Numbers in parentheses are t statistics, computed using White’s (1980) heteroscedasticity-consistent estimator for standard error. * denotes the estimate is significant at 1%.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Parameter Estimates</th>
<th>Parameter Estimates</th>
<th>Parameter Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sub-period 1</td>
<td>Sub-period 2</td>
<td>Sub-period 3</td>
</tr>
<tr>
<td></td>
<td>January 3, 1986 -</td>
<td>January 8, 1991 -</td>
<td>January 12, 1996 -</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>0.0008</td>
<td>0.1682</td>
<td>0.0021</td>
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<tr>
<td></td>
<td>(0.1067)</td>
<td>(0.0493)</td>
<td>(0.8750)</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>-0.0001</td>
<td>-0.0002</td>
<td>-0.0004</td>
</tr>
<tr>
<td></td>
<td>(-0.1250)</td>
<td>(-0.5000)</td>
<td>(-1.3333)</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>0.1747</td>
<td>0.0656</td>
<td>0.1594</td>
</tr>
<tr>
<td></td>
<td>(0.4810)</td>
<td>(0.0002)</td>
<td>(0.7756)</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>0.0919</td>
<td>0.0831</td>
<td>0.0148</td>
</tr>
<tr>
<td></td>
<td>(1.3054)</td>
<td>(1.6230)</td>
<td>(0.6115)</td>
</tr>
<tr>
<td>( h_1 )</td>
<td>0.0658*</td>
<td>0.0971*</td>
<td>0.0309*</td>
</tr>
<tr>
<td></td>
<td>(14.6222)</td>
<td>(16.1621)</td>
<td>(20.6000)</td>
</tr>
<tr>
<td>( h_2 )</td>
<td>0.0241*</td>
<td>0.0181*</td>
<td>0.0086*</td>
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<tr>
<td></td>
<td>(34.4286)</td>
<td>(90.5000)</td>
<td>(43.0000)</td>
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<tr>
<td>( c_1 )</td>
<td>0.5124*</td>
<td>0.6196*</td>
<td>0.5473*</td>
</tr>
<tr>
<td></td>
<td>(6.1293)</td>
<td>(11.4915)</td>
<td>(6.3173)</td>
</tr>
<tr>
<td>( d_{A,1} )</td>
<td>0.0048*</td>
<td>0.0666*</td>
<td>0.0207*</td>
</tr>
<tr>
<td></td>
<td>(4.4608)</td>
<td>(6.6587)</td>
<td>(4.5604)</td>
</tr>
<tr>
<td>( d_{B,1} )</td>
<td>0.0025*</td>
<td>0.0516*</td>
<td>0.0062*</td>
</tr>
<tr>
<td></td>
<td>(3.4567)</td>
<td>(5.1615)</td>
<td>(4.5334)</td>
</tr>
<tr>
<td>( d_{C,1} )</td>
<td>-0.0028*</td>
<td>-0.0138*</td>
<td>-0.0123*</td>
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<tr>
<td></td>
<td>(-9.4207)</td>
<td>(-5.1339)</td>
<td>(-3.0127)</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>1.1216*</td>
<td>1.0123*</td>
<td>0.9415*</td>
</tr>
<tr>
<td></td>
<td>(29.4306)</td>
<td>(453.1471)</td>
<td>(25.5343)</td>
</tr>
<tr>
<td>( d_{A,2} )</td>
<td>-0.0521*</td>
<td>-0.0128*</td>
<td>-0.1115*</td>
</tr>
<tr>
<td></td>
<td>(-3.7862)</td>
<td>(-4.4458)</td>
<td>(-6.0908)</td>
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<tr>
<td>( d_{B,2} )</td>
<td>-0.0132*</td>
<td>-0.0024*</td>
<td>-0.0179*</td>
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<tr>
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<td>(-4.6092)</td>
<td>(-8.8230)</td>
<td>(-4.7354)</td>
</tr>
<tr>
<td>( d_{C,2} )</td>
<td>0.0182*</td>
<td>0.0075*</td>
<td>0.0016*</td>
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<td></td>
<td>(5.2412)</td>
<td>(3.1208)</td>
<td>(5.2973)</td>
</tr>
</tbody>
</table>